MATH 147 QUIZ 6 SOLUTIONS

1. Calculate $\int_0^\pi \int_{\sin(x)}^{3\sin(x)} x(1+y)\,dy\,dx$ and sketch the domain of integration. (5 Points)

   We begin by computing the inner integral, after distributing the x. This gives

$$\int_0^\pi \int_{\sin(x)}^{3\sin(x)} x+xy\,dy\,dx = \int_o^\pi xy+xy^2/2\Big|_{\sin(x)}^{3\sin(x)} dx = \int_0^\pi \left(3x\sin(x)+9x\sin^2(x)/2 - x\sin(x) - x\sin^2(x)/2\right) dx.$$

   This simplifies down to $\int_0^\pi 2x\sin(x)+4x\sin^2(x)$. We compute each piece by parts. First, note that

$$\int_0^\pi x\sin(x)\,dx = -x\cos(x)\Big|_0^\pi - \int_0^\pi (-\cos(x))\,dx = -x\cos(x)+\sin(x)\Big|_0^\pi = \pi.$$

   Thus, the first term, $2\int_0^\pi x\sin(x)\ dx = 2\pi$. As for the second term, note that $\sin^2(x) = 1/2 - (1/2)\cos(2x)$, and so we have

$$\int_0^\pi x\sin^2(x)\,dx = 1/2\int_0^\pi x - x\cos(2x)\,dx.$$

   We then proceed with parts on the $x\cos(2x)$ term, giving us

$$\int_0^\pi x\cos(2x)\,dx = \frac{1}{2}x\sin(2x) - \int_0^\pi \frac{1}{2}\sin(2x)\,dx = 1/2\left[x\sin(2x)+1/2\cos(2x)\right]_0^\pi = 0.$$

   We finally see that $\frac{1}{2}\int_0^\pi x\ dx = x^2/4\big|_0^\pi = \pi^2/4$, so $4\int_0^\pi x\sin^2(x)\ dx = \pi^2$ Combining the two terms, we get $\int_0^\pi \int_{\sin(x)}^{3\sin(x)} x(1+y)\,dy\,dx = \pi^2+2\pi$.

2. Calculate $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (x^2+y^2)^2\,dx\,dy$. (5 points)

   The bounds suggest that we are integrating over a disc. Note that $-\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$ is equivalent to $x^2 \le 4-y^2$ or $x^2+y^2 \le 4$. That is, we are on a disc of radius 2. We also note that since $y \ge 0$, we are only on the top half of the disc. We make the standard polar transformation to obtain

$$\int_0^\pi \int_0^2 r^4 r\,dr\,d\theta = \int_0^\pi \left[\frac{r^6}{6}\right]_0^2 d\theta = \int_0^\pi 64/6\,d\theta = 64\pi/6.$$